1 Number Systems

Fastrack Revision

- **Natural Numbers (N):** Set of counting numbers.
  $N = \{1, 2, 3, 4, 5, \ldots\}$
- **Whole Numbers (W):** Set of natural numbers together with zero.
  $W = \{0, 1, 2, 3, 4, 5 \ldots\}$
- **Integers (Z):** Set of all whole numbers and negative of natural numbers.
  $Z = \{\ldots\ldots -5, -4, -3, -2, -1, 0, 1, 2, \ldots\}$
- **Rational Numbers (Q):** Numbers which can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
  1. Every integer, natural and whole number is a rational number.
  2. There are infinite rational numbers between two rational numbers.
  3. The sum, difference or product of two rational numbers is always a rational number. The quotient of a division of one rational number by a non-zero rational number is a rational number.
  4. The decimal expansion of a rational number is either terminating or non-terminating repeating (recurring).

     Let $\frac{p}{q}$ be the simplest form of a given number where p and q are integers and $q \neq 0$.

     (i) If $q = 2^m \times 5^n$ for some non-negative integers m and n, then $\frac{p}{q}$ is a terminating decimal.

     (ii) If $q \neq (2^m \times 5^n)$, then $\frac{p}{q}$ is a non-terminating repeating (recurring) decimal.
- **Equivalent Rational Numbers:** Two rational numbers are said to be equivalent, if numerator and denominators of both rational numbers are in proportion or they are reducible to be equal.
- **Irrational Numbers ($\overline{Q}$):** Numbers which cannot be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.
  1. The sum, difference, multiplication or division of two irrational numbers are not always irrational.
  2. The decimal expansion of an irrational number is non-terminating non-repeating (non-recurring).
- **Real Numbers (R):** Set of all rational and irrational numbers.
  Every real number is represented by a unique point on the number line. Also, every point on the number line represents a unique real number.
- **Radical Sign:** Suppose $a > 0$ be a real number and n be a positive integer. Then nth root of a is defined as $\sqrt[n]{a} = b$, if $b^n = a$ and $b > 0$. The symbol '$\sqrt{\ }$' used in $\sqrt[n]{a}$, i.e., $a^{\frac{1}{n}}$ is called the radical sign, where n and a are known as index and radicand respectively.

Knowledge BOOSTER

1. *The sum or difference of a rational number and an irrational number is irrational.*
2. *The product or quotient of a non-zero rational number with an irrational number is irrational.*
3. *A rational number between two rational numbers a and b is $\frac{a+b}{2}$.*
4. *Between two rational numbers a and b, n rational numbers are given by*
   $(a+d), (a+2d), (a+3d), \ldots, (a+nd)$, *where* $d = \frac{b-a}{n+1}$

- **Laws of Radicals:** Let a and b be positive real numbers. Then,
  1. $\sqrt{ab} = \sqrt{a}\sqrt{b}$
  2. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
  3. $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a - b$
  4. $(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$

or $(\sqrt{a}+b)(\sqrt{a}-b)=a-b^2$

5. $(\sqrt{a}+\sqrt{b})^2=a+2\sqrt{ab}+b$

6. $(\sqrt{a}+\sqrt{b})(\sqrt{c}+\sqrt{d})=\sqrt{ac}+\sqrt{ad}+\sqrt{bc}+\sqrt{bd}$

▸ **Rationalising Factors:** If a and b are positive numbers, then

1. Rationalising factor of $\frac{1}{\sqrt{a}}$ is $\sqrt{a}$.
2. Rationalising factor of $\frac{1}{a\pm\sqrt{b}}$ is $a\mp\sqrt{b}$.
3. Rationalising factor of $\frac{}{\sqrt{a}\pm\sqrt{b}}$ is $\sqrt{a}\mp\sqrt{b}$.

▸ **Laws of Exponents:** If a and b are positive real numbers and m and n are rational numbers, then

1. $a^m\times a^n=a^{m+n}$
2. $a^m\div a^n=a^{m-n}$, $m>n$
3. $(a^m)^n=a^{mn}$
4. $a^m b^m=(ab)^m$
5. $\frac{a^m}{b^m}=\left(\frac{a}{b}\right)^m$
6. $\left(\frac{a}{b}\right)^{-m}=\left(\frac{b}{a}\right)^m$
7. $(a^m)^{\frac{1}{n}}=a^{\frac{m}{n}}$
8. $a^{-m}=\frac{1}{a^m}$
9. $a^0=1$
10. $(\sqrt[n]{a})^m=a^{\frac{m}{n}}=\sqrt[n]{a^m}$

Practice Exercise

Multiple Choice Questions

Q 1. Every rational number is:
a. an integer
b. a whole number
c. a real number
d. a natural number

Q 2. Every real number is:
a. irrational
b. neither rational nor irrational
c. rational
d. either rational or irrational

Q 3. Between any two rational numbers, there:
a. is no rational number
b. is no irrational number
c. are exactly two rational numbers
d. are many rational numbers

Q 4. The sum of two irrational numbers is:
a. always an integer
b. always rational
c. always irrational
d. either irrational or rational

Q 5. What is the decimal representation of an irrational number?
a. Always terminating
b. Always non-terminating
c. Neither terminating nor repeating
d. Repeating

Q 6. Which number can neither be expressed as a terminating decimal nor as a repeating decimal?
a. An irrational number
b. A whole number
c. An integer
d. A rational number

Q 7. A rational number between 5 and 9 is:
a. 8
b. 7
c. 7.2
d. 7.5

Q 8. The rational number between $\sqrt{3}$ and $\sqrt{5}$ is:
a. 3.5
b. 2.1
c. 1.5
d. 4.8

Q 9. A rational number between $\sqrt{2}$ and $\sqrt{3}$ is:
a. $\frac{\sqrt{2}+\sqrt{3}}{2}$
b. $\frac{\sqrt{2}\times\sqrt{3}}{2}$
c. 1.8
d. 1.5

Q 10. If 'm' is a positive integer which is not a perfect square, then the value of $\sqrt{m}$ is:
a. a natural number
b. an irrational number
c. an integer
d. a rational number

Q 11. Which of the following is an irrational number?
a. 2.718
b. 2.717
c. 2.717171717171......
d. 2.7171171117......

Q 12. What is the decimal form of 2/11?
a. 0.18
b. 0.01823
c. 0.018
d. $0.\overline{18}$

Q 13. The value of 1.999 .... in the form $\frac{p}{q}$, where p and q are integers and $q\neq 0$ is:
a. $\frac{19}{20}$
b. $\frac{1999}{1000}$
c. 2
d. $\frac{1}{9}$

Q 14. The rationalisation factor of $3-\sqrt{7}$ is:
a. $\sqrt{7}-3$
b. $3+\sqrt{7}$
c. $\sqrt{3}-7$
d. $\sqrt{3}+7$

Q 15. If $x=7+4\sqrt{3}$, then the value of $\frac{1}{x}$ is:
a. $\frac{1}{7-4\sqrt{3}}$
b. $7-4\sqrt{3}$
c. $\frac{1}{7+4\sqrt{2}}$
d. None of these

Q 16. If $x=\sqrt{3}+11$, then $x-\frac{118}{x}$ is equal to:
a. 22
b. $-2\sqrt{3}$
c. $2\sqrt{3}$
d. $22+2\sqrt{3}$

Q 17. The simplified value of $\sqrt{5+2\sqrt{6}}$ is:

a. $\sqrt{3}+\sqrt{2}$ b. $\sqrt{3}-\sqrt{2}$

c. $\sqrt{2}-\sqrt{3}$ d. $2+\sqrt{3}$

Q 18. If $x+\sqrt{15}=4$, then $x+\frac{1}{x}$ is equal to:

a. $8+2\sqrt{15}$ b. $2\sqrt{15}$

c. $\sqrt{15}$ d. 8

Q 19. If $\sqrt{2}=1.4142$, then $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to:

a. 2.4142 b. 0.4142 c. 5.8282 d. 0.1718

Q 20. If $8^{x+1}=64$, then the value of x is:

a. 0 b. 2 c. 1 d. −1

Q 21. The simplified form of $\left(-\frac{1}{27}\right)^{-2/3}$ is:

a. $\frac{1}{9}$ b. − 9 c. 9 d. $-\frac{1}{9}$

Q 22. $\sqrt[4]{\sqrt[3]{2^2}}$ is equal to:

a. $2^{-\frac{1}{6}}$ b. $2^{\frac{1}{6}}$ c. 2^{-6} d. 2^6

Q 23. If $\left(\frac{2}{5}\right)^x\left(\frac{5}{2}\right)^{2x}=\frac{625}{16}$, then x is equal to:

a. 3 b. 4 c. − 4 d. 2

Q 24. If x is a positive real number and $x^2=11$, then x^3 is equal to:

a. 121 b. $\sqrt{11}$

c. $121\sqrt{11}$ d. $11\sqrt{11}$

Assertion & Reason Type Questions

Directions (Q. Nos. 25-29): *In the following questions, a statement of Assertion (A) is followed by a statement of a Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

Q 25. Assertion (A): Rational number lying between $\frac{1}{5}$ and $\frac{1}{3}$ is $\frac{4}{15}$.

Reason (R): Rational number lying between two rational numbers a and b is $\frac{a+b}{2}$.

Q 26. Assertion (A): 6.527 is a terminating decimal number.

Reason (R): Any decimal number is said to be a recurring decimal number, if set of digits is repeated periodically.

Q 27. Assertion (A): The rationalising factor of $8-\sqrt{7}$ is $8+\sqrt{7}$.

Reason (R): If the product of two irrational numbers is rational, then each one is said to be the rationalising factor of the other.

Q 28. Assertion (A): The simplified form of $7^4\times7^5$ is 7^{20}.

Reason (R): If $a>0$ be a real number and p and q be rational numbers. Then $a^p\times a^q=a^{p+q}$.

Q 29. Assertion (A): The sum of two irrational numbers $3-\sqrt{5}$ and $5+\sqrt{5}$ is rational number.

Reason (R): The sum of two irrational numbers is always an irrational number.

Fill in the Blanks Type Questions

Q 30. The product or quotient of a non-zero rational number with an irrational number is ....

Q 31. The rational numbers whose numerator and denominator both are equal or they are reducible to equal are called .... rational number.

Q 32. If the decimal representation of a number is non-terminating non-repeating, then the number is a/an .... number.

Q 33. $\sqrt[3]{(27)^{-2}}$ is equal to ....

Q 34. If $a=3$ and $b=5$, the value of a^a+b^b will be ....

True/False Type Questions

Q 35. The square roots of positive integers are always irrational.

Q 36. Every point on the number line is of the form $\sqrt{n}$, where n is a natural number.

Q 37. The number $(3-\sqrt{3})(3+\sqrt{3})$ is an irrational number.

Q 38. For rationalising the denominator of the expression $\frac{1}{\sqrt{12}}$ we multiply and divide by $\sqrt{2}$.

Q 39. If $a^{1/n}=\sqrt[n]{a}$, then symbol '$\sqrt{\ }$' is said to be a radical sign.

Solutions

1. (c) a real number
2. (d) either rational or irrational
3. (d) are many rational numbers.
4. (d) either irrational or rational
5. (c) Neither terminating nor repeating
6. (a) An irrational number
7. (b)

TR!CK

A rational number between a and b is $\frac{a+b}{2}$.

A rational number between 5 and 9 is

$$\frac{5+9}{2} = \frac{14}{2} = 7$$

8. (b) Since, $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$
Therefore the rational number between $\sqrt{3}$ and $\sqrt{5}$ (or 1.732 and 2.236) is 2.1 exist.

9. (d) Since, $\sqrt{2} = 1.414$ and $\sqrt{3} = 1.732$
Therefore the rational number between $\sqrt{2}$ and $\sqrt{3}$ (or 1.414 and 1.732) is 1.5 exist.

COMMON ERR!R

Some of the students write a rational number between $\sqrt{2}$ and $\sqrt{3}$ is $\frac{\sqrt{2}+\sqrt{3}}{2}$, which is wrong. So, please be careful for such types of problem.

10. (b) an irrational number
11. (d)

TiP

The decimal expansion of an irrational number is non-terminating non-repeating.

(a) 2.718 is a terminating decimal number so it is not an irrational number.

(b) 2.717 is a terminating decimal number so it is not an irrational number.

(c) 2.717171......... is a non-terminating repeating decimal number, so it is not an irrational number.

(d) 2.7171171117........ is a non-terminating non-repeating, so it is an irrational number.

12. (d)

$$\begin{array}{r|l} 11 & 2.0 \;(0.1818 \\ & \underline{11} \\ & 90 \\ & \underline{88} \\ & 20 \\ & \underline{11} \\ & 90 \\ & \underline{88} \\ & 2 \end{array}$$

$\therefore \frac{2}{11} = 0.1818$ ....

13. (c) Let $x = 1.999$......... ...(1)
Multiply both sides by 10, we get

$10x = 19.99$ ....(2)

Subtract eq. (1) from eq. (2), we get

$9x = 18 \Rightarrow x = 2$

14. (b) $3+\sqrt{7}$

15. (b) Given, $x = 7+4\sqrt{3}$

$\therefore \quad \frac{1}{x} = \frac{1}{7+4\sqrt{3}}$ ...(1)

The rational factor of $\frac{1}{x}$ is $7-4\sqrt{3}$. So, multiply numerator and denominator eq. (1) by $7-4\sqrt{3}$, we get

$$\frac{1}{x} = \frac{1}{7+4\sqrt{3}} \times \frac{7-4\sqrt{3}}{7-4\sqrt{3}}$$

TR!CK

$(a+b)(a-b) = a^2 - b^2$

$$\Rightarrow \quad \frac{1}{x} = \frac{7-4\sqrt{3}}{(7)^2-(4\sqrt{3})^2}$$

$$= \frac{7-4\sqrt{3}}{49-48} = \frac{7-4\sqrt{3}}{1}$$

$$= 7-4\sqrt{3}$$

16. (c) Given, $x = \sqrt{3}+11$

$$\therefore \quad \frac{1}{x} = \frac{1}{\sqrt{3}+11} \times \frac{\sqrt{3}-11}{\sqrt{3}-11}$$

$$= \frac{\sqrt{3}-11}{(\sqrt{3})^2-(11)^2}$$

$$= \frac{\sqrt{3}-11}{3-121}$$

$$= \frac{\sqrt{3}-11}{-118} = \frac{11-\sqrt{3}}{118}$$

$$\therefore \quad x - \frac{118}{x} = (\sqrt{3}+11) - 118\left(\frac{11-\sqrt{3}}{118}\right)$$

$$= \sqrt{3}+11-(11-\sqrt{3})$$

$$= 2\sqrt{3}$$

17. (a) $\sqrt{5+2\sqrt{6}} = \sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2\sqrt{3}\cdot\sqrt{2}}$

$$= \sqrt{(\sqrt{3}+\sqrt{2})^2}$$

$$= \sqrt{3}+\sqrt{2}$$

18. (d) Given, $x+\sqrt{15} = 4$

or $\quad x = 4-\sqrt{15}$

$$\frac{1}{x} = \frac{1}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}}$$

TR!CK

$(a+\sqrt{b})(a-\sqrt{b}) = a^2 - b$

$$= \frac{4+\sqrt{15}}{(4)^2 - 15} = \frac{4+\sqrt{15}}{16-15} = 4+\sqrt{15}$$

$$\therefore \quad x + \frac{1}{x} = 4 - \sqrt{15} + 4 + \sqrt{15}$$

$$= 8$$

19. (b) Given, $\sqrt{2} = 1.4142$

$$\therefore \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}}$$

$$= \sqrt{\frac{(\sqrt{2}-1)}{(\sqrt{2})^2 \quad (1)^2}} = \sqrt{2}-1$$

$$= \sqrt{2} - 1$$

$$= 1.4142 - 1 = 0.4142$$

20. (c) Given, $8^{x+1} = 64$

$$\therefore \quad 8^{x+1} = 8^2$$

Compare exponent both sides, we get

$x + 1 = 2 \Rightarrow x = 1$

21. (c) $\left(-\frac{1}{27}\right)^{-2/3} = (-27)^{2/3}$ $\left[\because \left(\frac{1}{p}\right)^{-q} = p^q\right]$

$$= (-1)^{2/3}(27)^{2/3}$$

$$= ((-1)^2)^{\frac{1}{3}}(3^3)^{\frac{2}{3}}$$

$$= (1)^{1/3}(3)^{3\times\frac{2}{3}}$$

$$= 1\,(3)^2$$

$$= 9$$

22. (b) $\sqrt[4]{\sqrt[3]{2^2}} = \sqrt[4]{(2^2)^{\frac{1}{3}}}$

$$= \left(2^{\frac{2}{3}}\right)^{\frac{1}{4}}$$

$$= 2^{\frac{1}{6}}$$

23. (b) Given, $\left(\frac{2}{5}\right)^x \left(\frac{5}{2}\right)^{2x} = \frac{625}{16}$

$$\Rightarrow \left(\frac{5}{2}\right)^{-x}\left(\frac{5}{2}\right)^{2x} = \left(\frac{5}{2}\right)^4$$

$$\Rightarrow \left(\frac{5}{2}\right)^{2x-x} = \left(\frac{5}{2}\right)^4$$

$$\Rightarrow \left(\frac{5}{2}\right)^{x} = \left(\frac{5}{2}\right)^4$$

Compare exponent both sides, we get

$$x = 4$$

24. (d) Given, $x^2 = 11$

$$\Rightarrow \quad x = (11)^{1/2}$$

Cubing both sides, we get

$$x^3 = (11)^{3/2}$$

$$= 11\sqrt{11}$$

25. (a) **Assertion (A):** Rational number lying between

$\frac{1}{5}$ and $\frac{1}{3}$ is $\frac{\frac{1}{5}+\frac{1}{3}}{2} = \frac{3+5}{} = \frac{8}{30}$

$$= \frac{4}{15}$$

So, Assertion (A) is true.

Reason (R): It is true to say that rational number lying between two rational numbers a and b is $\frac{a+b}{2}$.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

26. (b) **Assertion (A):** In decimal number 6.527, their is no set of digits is repeated, so it is a terminating decimal number.

So, Assertion (A) is true.

Reason (R): It is true to say that any decimal number is said to be a recurring decimal number, if set of digits is repeated periodically.

Hence, both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

27. (a) **Assertion (A):** It is true that the rationalising factor of $8-\sqrt{7}$ is $8+\sqrt{7}$.

Reason (R): It is true to say that each one is rationalising factor in the product of two irrational numbers.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

28. (d) **Assertion (A):** $7^4 \times 7^5 = (7)^{4+5} = (7)^9$

So, Assertion (A) is false.

Reason (R): It is true to say that $a^p \times a^q = a^{p+q}$

Hence, Assertion (A) is false but Reason (R) is true.

29. (c) **Assertion (A):** Here, $3-\sqrt{5}+5+\sqrt{5} = 8$, which is a rational number.

So, Assertion (A) is true.

Reason (R): It is not always true to say that sum of two irrational number is always an irrational number.

Hence, Assertion (A) is true but Reason (R) is false.

30. irrational

31. equivalent

32. irrational

33. $\sqrt[3]{(27)^{-2}} = \sqrt[3]{(3^3)^{-2}}$

$= (3^{-3\times2})^{\frac{1}{3}}$

$= 3^{-2} = \frac{1}{9}$

34. Given, $a = 3$ and $b = 5$

$\therefore\ a^a + b^b = (3)^3 + (5)^5$

$= 27 + 3125$

$= 3152$

35. False

36. False

37. False

$(3-\sqrt{3})(3+\sqrt{3}) = (3)^2 - (\sqrt{3})^2$

$= 9 - 3$

$= 6$, which is rational.

38. False

$$\frac{1}{\sqrt{12}} = \frac{1}{\sqrt{3\times4}} = \frac{1}{2\sqrt{3}}$$

To rationalise the denominator, we multiply numerator and denominator by $\sqrt{3}$.

39. True

Case Study Based Questions

Case Study 1

During revision hours, two students Vimal and Sunil were discussing with each other about the topic of rationalising the denominator.

Vimal explains that simplification of $\frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ by rationalising the denominator is multiplying numerator and denominator by $\sqrt{5}-\sqrt{3}$.

And Sunil explains the simplification of $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$ by using the identity $(a+b)\,(a-b) = a^2 - b^2$.

On the basis of the above information, solve the following questions:

Q 1. The rationalising factor of $\sqrt{5}+\sqrt{3}$ is:

a. $\frac{1}{\sqrt{5}+\sqrt{3}}$ b. $(\sqrt{5}-\sqrt{3})$

c. $-\sqrt{5}-\sqrt{3}$ d. $(\sqrt{5}+\sqrt{3})$

Q 2. According to Vimal explanation, the simplification of $\frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ is:

a. $5-\sqrt{15}$ b. $\frac{5+\sqrt{15}}{2}$

c. $\frac{\sqrt{15}-3}{2}$ d. $5+\sqrt{15}$

Q 3. According to Sunil explanation, the simplification of $(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})$ is:

a. 1 b. -1 c. 2 d. 3

Q 4. Addition of two irrational numbers:

a. is always rational

b. is always irrational

c. may be rational or irrational

d. is always integer

Q 5. The square root of natural number is a/an:

a. rational b. irrational

c. rational or irrational d. None of these

Solutions

1. (b) The rationalising factor of $(\sqrt{5}+\sqrt{3})$ is $(\sqrt{5}-\sqrt{3})$.

So, option (b) is correct.

2. (c) We have, $\frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

By rationalisation the denominator

$$\frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}$$

$$= \frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{\sqrt{3}(\sqrt{5}-\sqrt{3})}{5-3} = \frac{\sqrt{15}-3}{2}$$

So, option (c) is correct.

3. (a) Using identity $(a+b)\,(a-b) = a^2 - b^2$

$\therefore\ (\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) = (\sqrt{3})^2 - (\sqrt{2})^2$

$= 3 - 2 = 1$

So, option (a) is correct.

4. (c) Addition of two irrational numbers may be rational or irrational.

e.g. (i) $(2+\sqrt{5})+(1-\sqrt{5}) = 3$, which is rational.

(ii) $(2+\sqrt{5})+\sqrt{3} = 2+\sqrt{5}+\sqrt{3}$,

which is irrational.

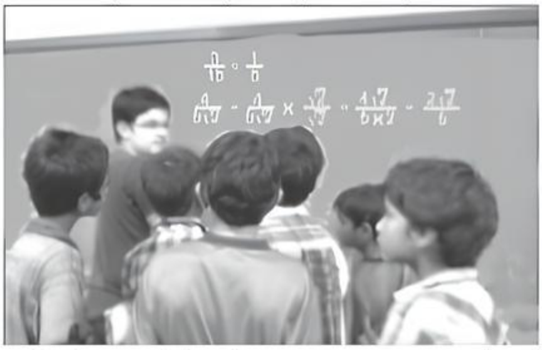
So, option (c) is correct.

5. (c) The square root of a natural number is a rational or irrational number.
So, option (c) is correct.

Case Study 2

One day a math teacher taught students about the number system. She drew a number line on the black board and represented different types of numbers such as natural numbers, integers, rational numbers, etc.

A number of the form $\frac{p}{q}$ is said to be a rational number, if $q \neq 0$ and p and q are integers.



On the basis of the above information, solve the following questions:

Q 1. A rational number between $\frac{1}{3}$ and $\frac{1}{7}$ is:

a. $\frac{21}{5}$ b. $\frac{17}{21}$ c. $-\frac{5}{21}$ d. $\frac{5}{21}$

Q 2. An irrational number between $\sqrt{3}$ and $\sqrt{5}$ is:

a. 2.1 b. 2
c. $\sqrt{3.5}$ d. $\sqrt{7}$

Q 3. Decimal number $1.\overline{5}$ in the form of $\frac{p}{q}$ is:

a. $\frac{14}{3}$ b. $\frac{11}{9}$ c. $-\frac{14}{9}$ d. $\frac{14}{9}$

Q 4. The sum of two rational numbers is always:

a. integers b. naturals
c. rational d. irrational

Q 5. A terminating or repeating decimal number is a/an:

a. rational b. irrational
c. rational or irrational d. whole number

Solutions

1. (d) A rational number between $\frac{1}{3}$ and $\frac{1}{7}$ is

$$\frac{\frac{1}{3}+\frac{1}{7}}{2} = \frac{7+3}{2\times21} = \frac{10}{2\times21} = \frac{5}{21}$$

So, option (d) is correct.

2. (c) Since, $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$
But $\sqrt{3.5} = 1.870$,which lies in the given interval.
Hence, irrational number $\sqrt{3.5}$ lies between $\sqrt{3}$ and $\sqrt{5}$.
So, option (c) is correct.

3. (d) Let $x = 1.\overline{5}$
$\Rightarrow \quad x = 1.555.........$ ...(1)
Multiplying both sides by 10, we get
$10x = 15.55.........$ ...(2)
Subtracting eq. (1) from eq. (2), we get
$9x = 14 \Rightarrow x = \frac{14}{9}$
So, option (d) is correct.

4. (c) The sum of two rational numbers is always a rational number.
So option (c) is correct.

5. (a) A terminating or repeating decimal number is a rational number.
So, option (a) is correct.

Case Study 3

The secretary of Golf Course colony organised a free medical camp for the patients suffering from Corona Virus. During the medical camp, the pulse rate of many patients recorded in mathematical terms was 150.35, $\frac{174}{180}$, $(178)^{1/2}$, ... But it was not easy to understand the mathematical number. So one of the student of class IX asked the following questions.

On the basis of the above information, solve the following questions:

Q 1. What is the value of $2.1\overline{3}$?

a. $\frac{203}{90}$ b. $\frac{211}{90}$
c. $\frac{211}{99}$ d. $-\frac{211}{99}$

Q 2. The value of $\sqrt{18}\times\sqrt{10}\times\sqrt{5}$ is:

a. 25 b. 35 c. 32 d. 30

Q 3. Rational number $\frac{22}{7}$ is equivalent to:

a. π b. 3.142857

c. $3.14\overline{2857}$ d. $3.\overline{142857}$

Q 4. If $4^{2x} = \frac{1}{8}$, then the value of x is:

a. $\frac{3}{4}$ b. $-\frac{3}{4}$ c. $-\frac{4}{3}$ d. $\frac{4}{3}$

Q 5. Which one of the following numbers is rational?

a. $\sqrt{5}$ b. π c. $\frac{7}{0}$ d. $\frac{0}{7}$

Solutions

1. (c) Let $x = 2.\overline{13}$

$\Rightarrow$ $x = 2.1313......$ ...(1)

Multiplying both sides by 100, we get

$100x = 213.1313......$ ...(2)

Subtracting eq. (1) from eq. (2), we get

$99x = 211$

$\Rightarrow$ $x = \frac{211}{99}$

So, option (c) is correct.

2. (d) $\sqrt{18} \times \sqrt{10} \times \sqrt{5}$

$= \sqrt{2 \times 3 \times 3} \times \sqrt{2 \times 5} \times \sqrt{5}$

$= \sqrt{2 \times 2 \times 3 \times 3 \times 5 \times 5} = 2 \times 3 \times 5 = 30$

So, option (d) is correct.

3. (d) $\frac{22}{7} = 3.142857142..........$

$= 3.\overline{142857}$

```
7) 22(3.142857142
   21
    10
     7
     30
     28
      20
      14
       60
       56
        40
        35
         50
         49
          10
           7
          30
          28
           20
           14
            6
```

So, option (d) is correct.

4. (b) We have, $4^{2x} = \frac{1}{8}$

$\therefore$ $(2^2)^{2x} = \frac{1}{2^3}$

$\Rightarrow$ $2^{4x} \times 2^3 = 1$

$\Rightarrow$ $2^{4x+3} = 2^0$

Compare the exponent, we get

$4x + 3 = 0 \Rightarrow x = -\frac{3}{4}$

So, option (b) is correct.

5. (d) (a) $\sqrt{5}$ is an irrational number.

(b) π is an irrational number.

(c) $\frac{7}{0}$ is not defined.

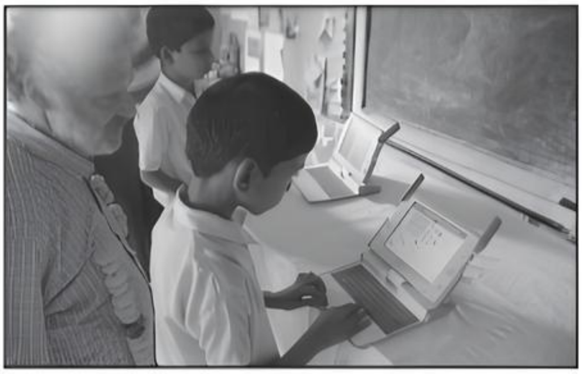
(d) $\frac{0}{7} = 0$, it is a rational number.

So, option (d) is correct.

Case Study 4

Mrs. Rakhi lives in an undeveloped area where there is no facility of proper education. But one thing is available in that area *i.e.*, network. Since she was very keen to take education, so she decided to complete her education through e-learning.

One day she was studying number system, where she learnt about rational numbers, irrational numbers and decimal numbers, etc.



On the basis of the above information, solve the following questions:

Q 1. Convert the rational number $\frac{2}{15}$ into decimal number.

Q 2. Write one irrational number between 2.365 and 3.125.

Q 3. If $x + \sqrt{2} = 3$, then find the value of $\frac{1}{x}$.

Q 4. Find the product of two irrational numbers $(7 + 3\sqrt{2})$ and $(7 - 3\sqrt{2})$.

Solutions

1. $\frac{2}{15} = 0.1333........$

$= 0.1\overline{3}$

```
15)2.0(0.133
   15
   50
   45
    50
    45
     5
```

2. One irrational number between 2.365 and 3.125 is 2.6121121112........ .

3. We have, $x + \sqrt{2} = 3$

$\Rightarrow \quad x = 3 - \sqrt{2}$

$\because \quad \frac{1}{x} = \frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}}$ [by rationalisation]

$= \frac{3+\sqrt{2}}{(3)^2 - (\sqrt{2})^2}$

$= \frac{3+\sqrt{2}}{9-2} = \frac{3+\sqrt{2}}{7}$

4. $(7+3\sqrt{2})(7-3\sqrt{2}) = (7)^2 - (3\sqrt{2})^2$

$= 49 - 18$

$= 31$

Case Study 5

In a coaching centre, one day a teacher told student about the laws of exponents, which was defined below:

Suppose $a > 0$, $b > 0$ be real numbers and let m and n be rational numbers. Then

(i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$ (iv) $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

(v) $(ab)^m = a^m b^m$

On the basis of the above information, solve the following questions:

Q 1. Find the value of $2^3 \times 2^4$.

Q 2. If $8^{x+1} = 64$, then find the value of x.

Q 3. Find the value of $\left((81)^{1/2}\right)^{1/2}$.

Q 4. If $x = 0.000216$, then find the value of $(x)^{1/3}$.

Q 5. Simplify $(49)^{1/3} \times (7)^{1/3}$.

Solutions

1. $2^3 \times 2^4 = (2)^{3+4} = 2^7$

$= 128$

2. Given, $8^{x+1} = 64$

$\therefore \quad 8^{x+1} = 8^2$

Compare the exponents, we get

$x + 1 = 2$

$\Rightarrow \quad x = 1$

3. $\left((81)^{1/2}\right)^{1/2} = (3^4)^{\frac{1}{4}}$

$= 3^{4 \times \frac{1}{4}} = 3^1$

$= 3$

4. Given, $x = 0.000216$

$= \frac{216}{1000000} = \left(\frac{6}{100}\right)^3$

$\therefore \quad (x)^{1/3} = \left(\left(\frac{6}{100}\right)^3\right)^{1/3} = \left(\frac{6}{100}\right)^{3 \times \frac{1}{3}} = \frac{6}{100}$

$= 0.06$

5. $(49)^{1/3} \times (7)^{1/3} = (7^2)^{1/3} \times (7)^{1/3}$

$= 7^{\frac{2}{3}+\frac{1}{3}} = 7^{\frac{3}{3}} = 7^1 = 7$

Very Short Answer Type Questions

Q 1. Is $\sqrt{\frac{50}{2}}$ a rational number or not?

Q 2. Find a rational number between $\frac{-3}{7}$ and $\frac{1}{3}$.

Q 3. Convert the following decimal numbers into form $\frac{m}{n}$.

(i) 0.35 (ii) 0.175

Q 4. Write the rational number $\frac{43}{100}$ in decimal form and what kind of rational number it has?

Q 5. Calculate the value of $2.\overline{9}$ in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Q 6. Write the two irrational numbers between 2 and 2.5.

Q 7. Simplify $\sqrt{72}+\sqrt{800}-\sqrt{18}$.

Q 8. Simplify $(3+\sqrt{5})(3-\sqrt{5})$ by using identity.

Q 9. Write the rationalising factor of $\frac{1}{5+\sqrt{3}}$.

Q 10. Simplify $(\sqrt{3}+\sqrt{7})^2$.

Q 11. Write the simplified value of $(25)^{-\frac{1}{4}} \div (25)^{\frac{1}{4}}$.

Q 12. Find the value of $\left(\frac{1}{125}\right)^{-\frac{2}{3}} \div \left(\frac{1}{216}\right)^{-\frac{4}{3}}$.

Q 13. Simplify $\{(-2)^0+(5)^0+(-13)^0\}^2$.

Short Answer Type-I Questions

Q 1. Is zero (0) a rational number? Justify your answer.

Q 2. Insert three rational numbers between $\frac{3}{5}$ and $\frac{5}{7}$.

Q 3. Classify the following numbers as rational or irrational with justification:

(i) $\sqrt{144}$ (ii) $\sqrt{\frac{9}{27}}$

(iii) 10.124124... (iv) 1.010010001...

Q 4. Express $23.\overline{43}$ in $\frac{p}{q}$ form, where p, q are integers and $q \neq 0$.

Q 5. Find two irrational numbers between $\frac{1}{7}$ and $\frac{2}{7}$ when it is given that $\frac{1}{7} = 0.142857142857\ldots$.

Q 6. If $\sqrt{5}=2.236$, then find the approximate value of $\frac{3}{\sqrt{5}}$.

Q 7. If $x = 3+2\sqrt{2}$, check whether $x+\frac{1}{x}$ is rational or irrational.

Q 8. Find the value of a and b, if $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$.

Q 9. If $x=1+\sqrt{2}$, then find the value of $\left(x-\frac{1}{x}\right)^2$.

Q 10. Taking $\sqrt{2} = 1.414$ and $\pi = 3.14$, evaluate $\frac{1}{\sqrt{2}}+\pi$ upto three places of decimal.

Q 11. Simplify that $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}$.

Q 12. If $a = 2$ and $b = 3$, then find the value of: (i) $(a^b+b^a)^{-1}$ (ii) $(a^a+b^b)^{-1}$

Q 13. Show that $\frac{x^{p(q-r)}}{x^{q(p-r)}} \div \left(\frac{x^q}{x^p}\right)^r = 1$.

Q 14. Simplify $\frac{5^{36}+5^{35}+5^{34}}{5^{32}+5^{31}+5^{30}}+\frac{3^{40}+3^{39}+3^{38}}{3^{41}+3^{40}+3^{39}}$

Short Answer Type-II Questions

Q 1. Find six rational numbers between 3 and 4.

Q 2. Express $4.0\overline{35}$ in form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Q 3. Evaluate $\sqrt{5+2\sqrt{6}}+\sqrt{8-2\sqrt{15}}$.

Q 4. Rationalise the denominator of $\frac{\sqrt{3}+\sqrt{2}}{5+\sqrt{2}}$.

Q 5. If $a = \frac{\sqrt{2}+1}{\sqrt{2}-1}$ and $b = \frac{\sqrt{2}-1}{\sqrt{2}+1}$, then the value of a^2+b^2-4ab.

Q 6. If $\frac{30}{4\sqrt{3}+3\sqrt{2}} = 4\sqrt{3}-a\sqrt{2}$, find the value of a.

Q 7. Show how $\sqrt{13}$ can be represented on the number line?

Q 8. Locate $\sqrt{10}$ on the number line.

Q 9. Represent $\sqrt{6.5}$ on the number line.

Q 10. Simplify $\frac{9^{\frac{1}{3}} \times 27^{\frac{-1}{2}}}{3^{\frac{1}{6}} \times 3^{\frac{-2}{3}}}$.

Q 11. Find x, if $\left(\frac{2}{3}\right)^x \cdot \left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$.

Q 12. Evaluate $\frac{1}{3}(\sqrt{7})^6 \times (25)^{\frac{3}{2}} \times \left(\frac{1}{5^3}\right)$.

Q 13. Write $\sqrt[3]{4}, \sqrt{3}, \sqrt[4]{6}$ in ascending order.

Q 14. If $\frac{9^n \times 3^2 \times (3)^{-\frac{n}{2} \times -2} - (27)^n}{3^{3m} \times 2^3} = \frac{1}{27}$, prove that $m-n=1$.

Q 15. Show that $\frac{1}{1+x^{a-b}}+\frac{1}{1+x^{b-a}} = 1$.

Long Answer Type Questions

Q 1. Give two rational numbers whose:

(i) sum is a rational number.

(ii) difference is a rational number.

(iii) product is a rational number.

(iv) division is a rational number.

Justify also.

Q 2. Express $1.3\overline{2} + 0.\overline{35}$ as a fraction in the simplest form.

Q 3. Let x be rational and y be irrational. Is xy necessarily irrational? Justify your answer by an example.

Q 4. Prove that

$$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots + \frac{1}{\sqrt{8}+3} = 2$$

Q 5. Find a and b, if

$$\frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}-\sqrt{3}} + \frac{2\sqrt{5}-\sqrt{3}}{2\sqrt{5}+\sqrt{3}} = a + \sqrt{15}\,b.$$

Q 6. Simplify $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$.

Q 7. If $x = \frac{1}{2-\sqrt{3}}$, then find the value of $x^3 - 2x^2 - 7x + 5$.

Q 8. If $x = \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}}$, then prove that $qx^2 - 2px + q = 0$, where $q \neq 0$.

Q 9. If $a = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ and $b = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$. Find the value of $\frac{a^2+ab+b^2}{a^2-ab+b^2}$.

Q 10. Find the value of

$$\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}.$$

Q 11. Simplify $\left(\frac{2^{-1}\times 3^2}{2^2\times 3^{-4}}\right)^{\frac{7}{2}} \times \left(\frac{2^{-2}\times 3^3}{2^3\times 3^{-5}}\right)^{\frac{-5}{2}}$.

Q 12. If $a = \frac{2^{x-1}}{2^{x-2}}, b = \frac{2^{-x}}{2^{x+1}}$ and $a - b = 0$, find the value of x.

Q 13. If $5^{2x-1} - 25^{x-1} = 2500$, then find the value of x.

Solutions

Very Short Answer Type Questions

1. $\sqrt{\frac{50}{2}} = \sqrt{25} = 5$

   So, it is a rational number.

2. A rational number between $\frac{-3}{7}$ and $\frac{1}{3}$

$$= \frac{\frac{-3}{7}+\frac{1}{3}}{2} = \frac{1}{2}\left(\frac{-9+7}{21}\right)$$

$$= \frac{1}{2}\left(\frac{-2}{21}\right) = \frac{-1}{21}$$

3. (i) $0.35 = \frac{35}{100} = \frac{5\times 7}{5^2\times 2^2}$

$$= \frac{7}{5\times 2^2} = \frac{7}{5\times 4} = \frac{7}{20}$$

   (ii) $0.175 = \frac{175}{1000} = \frac{5^2\times 7}{5^3\times 2^3}$

$$= \frac{7}{5\times 2^3} = \frac{7}{5\times 8} = \frac{7}{40}$$

4. $\frac{43}{100} = 0.43$, which is a terminating decimal expansion.

5. Let $x = 2.\overline{9} = 2.9999......$ ...(1)

   Multiplying both sides by 10, we get

   $10x = 29.999......$ ...(2)

   Subtracting eq. (1) from eq. (2), we get

   $10x - x = (29.999...) - (2.999...)$

$\Rightarrow \quad 9x = 27$

$\Rightarrow \quad x = \frac{27}{9} = 3$

6. The two irrational numbers between 2 and 2.5 can be taken as 2.101001000100001... and 2.201001000100001...

   Alternate Method: If a and b are two distinct positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b.

   $\therefore$ An irrational number between 2 and 2.5

$$= \sqrt{2\times(2.5)} = \sqrt{5}$$

   Similarly, an irrational number between 2 and $\sqrt{5}$

$$= \sqrt{2\sqrt{5}}$$

   Hence, two irrational numbers between 2 and 2.5 are $\sqrt{5}$ and $\sqrt{2\sqrt{5}}$.

7. $\sqrt{72} + \sqrt{800} - \sqrt{18}$

$$= \sqrt{36\times 2} + \sqrt{400\times 2} - \sqrt{9\times 2}$$

$$= 6\sqrt{2} + 20\sqrt{2} - 3\sqrt{2} = 23\sqrt{2}$$

8.

TR!CK

Using identity $(a+b)(a-b) = a^2 - b^2$

$$\therefore \quad (3+\sqrt{5})(3-\sqrt{5}) = (3)^2 - (\sqrt{5})^2$$

$$= 9 - 5 = 4$$

9. $\frac{1}{5+\sqrt{3}} = \frac{1}{5+\sqrt{3}} \times \frac{5-\sqrt{3}}{5-\sqrt{3}}$ (Rationalising the denominator)

$$= \frac{5-\sqrt{3}}{(5)^2-(\sqrt{3})^2} = \frac{5-\sqrt{3}}{25-3} = \frac{5-\sqrt{3}}{22}$$

Hence, the rationalising factor is $\frac{5-\sqrt{3}}{22}$.

10.

TR!CK

Using identity $(a+b)^2 = a^2+b^2+2ab$

$\therefore (\sqrt{3}+\sqrt{7})^2 = (\sqrt{3})^2 + (\sqrt{7})^2 + 2\sqrt{3}\sqrt{7}$

$= 3+7+2\sqrt{21}$

$= 10+2\sqrt{21}$

11. $(25)^{-\frac{1}{4}} \div (25)^{\frac{1}{4}} = (25)^{-\frac{1}{4}} \times (25)^{-\frac{1}{4}}$

$= (25)^{-\frac{1}{4}-\frac{1}{4}} = (25)^{-\frac{1}{2}}$

$= \frac{1}{(25)^{1/2}} = \frac{1}{(5^2)^{1/2}} = \frac{1}{5}$

$\left(\because \frac{a^m}{a^n} = a^{m-n}; a^{-n} = \frac{1}{a^n}\right)$

12. $\left(\frac{1}{125}\right)^{-\frac{2}{3}} \div \left(\frac{1}{216}\right)^{-\frac{4}{3}} = (125)^{\frac{2}{3}} \div (216)^{\frac{4}{3}}$

$= 5^{3\times\frac{2}{3}} \div 6^{3\times\frac{4}{3}} = 5^2 \div 6^4 = \frac{25}{1296}$

13. $((-2)^0 + (5)^0 + (-13)^0)^2$

$= (1+1+1)^2 \qquad (\because a^0 = 1)$

$= (3)^2 = 9$

Short Answer Type-I Questions

1. Yes, zero is a rational number.

Zero can be expressed as $\frac{0}{5}, \frac{0}{15}, \frac{0}{27}, \frac{0}{100}$, etc.

which are in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

2.

TR!CK

Between two rational numbers a and b, n rational numbers are given by

$(a+d), (a+2d), \ldots\ldots, (a+nd)$

where, $d = \frac{b-a}{n+1}$.

Here, $a = \frac{3}{5}, \quad b = \frac{5}{7}, \quad n = 3$

and $d = \frac{b-a}{n+1} = \frac{\frac{5}{7}-\frac{3}{5}}{3+1}$

$= \frac{25-21}{7\times5\times4} = \frac{4}{7\times5\times4} = \frac{1}{35}$

$a+d = \frac{3}{5}+\frac{1}{35} = \frac{21+1}{35} = \frac{22}{35}$

$a+2d = \frac{3}{5}+\frac{2}{35} = \frac{21+2}{35} = \frac{23}{35}$

and $a+3d = \frac{3}{5}+\frac{3}{35} = \frac{21+3}{35} = \frac{24}{35}$

Hence, three rational numbers between $\frac{3}{5}$ and $\frac{5}{7}$ are $\frac{22}{35}, \frac{23}{35}$ and $\frac{24}{35}$.

Alternate Method

LCM of 5 and 7 is 35.

$\therefore \quad \frac{3}{5} = \frac{3\times7}{5\times7} = \frac{21}{35}$

and $\quad \frac{5}{7} = \frac{5\times5}{7\times5} = \frac{25}{35}$

So, $\quad \frac{21}{35} < \frac{22}{35} < \frac{23}{35} < \frac{24}{35} < \frac{25}{35}$.

Hence, the required three rational numbers are $\frac{22}{35}, \frac{23}{35}$ and $\frac{24}{35}$.

3. (i) $\sqrt{144} = \sqrt{(12)^2} = 12$

Hence, it is a rational number.

(ii) $\sqrt{\frac{9}{27}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$

Since, $\sqrt{3}$ is an irrational number, so $\frac{1}{\sqrt{3}}$ is an irrational number.

(iii) 10.124124... is a non-terminating recurring decimal number, hence it is a rational number.

(iv) 1.010010001... is a number with non-terminating non-recurring decimal number, hence it is an irrational number.

4. Let $\quad x = 23.\overline{43}$

$\Rightarrow \quad x = 23.4343\ldots \qquad \ldots(1)$

Multiplying eq. (1) by 100, we get

$100x = 2343.4343\ldots \qquad \ldots(2)$

Subtracting eq. (1) from eq. (2), we get

$99x = 2320$

$\Rightarrow \quad x = \frac{2320}{99}$

Hence, $\quad 23.\overline{43} = \frac{2320}{99}$

5. We have, $\frac{1}{7} = 0.\overline{142857}$

and $\quad \frac{2}{7} = 0.\overline{285714}$

Both these rational numbers have non-terminating repeating decimals.

Now irrational number between $\frac{1}{7}$ and $\frac{2}{7}$ should have a non-terminating non-repeating expansion.

Hence, two irrational numbers between $\frac{1}{7}$ and $\frac{2}{7}$ may be 0.1430143001430001... and 0.254025400254...

6. Given, $\sqrt{5} = 2.236$

$\therefore \frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ (Rationalising the denominator)

$= \frac{3 \times \sqrt{5}}{5}$

$= \frac{3}{5} \times 2.236$

$= \frac{6.708}{5} = 1.3416$

7. Given, $x = 3 + 2\sqrt{2}$

Now, $\frac{1}{x} = \frac{1}{3+2\sqrt{2}} = \frac{1}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

$= \frac{3-2\sqrt{2}}{(3)^2-(2\sqrt{2})^2}$

(Rationalising the denominator)

$= \frac{3-2\sqrt{2}}{9-8} = 3 - 2\sqrt{2}$

$\therefore x + \frac{1}{x} = (3+2\sqrt{2}) + (3-2\sqrt{2}) = 6,$

which is a rational number.

8. Given, $\frac{3+\sqrt{2}}{3-\sqrt{2}} = a + b\sqrt{2}$

Rationalising the denominator of LHS

$\frac{3+\sqrt{2}}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = a + b\sqrt{2}$

$\Rightarrow \frac{(3+\sqrt{2})^2}{(3)^2-(\sqrt{2})^2} = a + b\sqrt{2}$

$\Rightarrow \frac{9+2+6\sqrt{2}}{9-2} = a + b\sqrt{2}$

TR!CK

$a^2 - b^2 = (a+b)(a-b); (a+b)^2 = a^2 + b^2 + 2ab$

$\Rightarrow \frac{11}{7} + \frac{6\sqrt{2}}{7} = a + b\sqrt{2}$

Comparing coefficients on both sides, we get

$a = \frac{11}{7}$ and $b = \frac{6}{7}$

9. Given $x = 1 + \sqrt{2}$

Now, $\frac{1}{x} = \frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$ (By rationalisation)

$= \frac{1-\sqrt{2}}{(1)^2 - (\sqrt{2})^2}$

$= \frac{1-\sqrt{2}}{1-2} = \frac{1-\sqrt{2}}{-1}$

$= \sqrt{2} - 1$

$\therefore \left(x - \frac{1}{x}\right)^2 = \left(1 + \sqrt{2} - (\sqrt{2}-1)\right)^2$

$= (2)^2 = 4$

10. $\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ (Rationalising the denominator)

$= \frac{\sqrt{2}}{2} = \frac{1.414}{2} = 0.707$

$\therefore \frac{1}{\sqrt{2}} + \pi = 0.707 + 3.14 = 3.847$

11. $\sqrt[4]{81} - 8\sqrt[3]{216} + 15\sqrt[5]{32} + \sqrt{225}$

$= (3^4)^{\frac{1}{4}} - 8(6^3)^{\frac{1}{3}} + 15(2^5)^{\frac{1}{5}} + (15^2)^{\frac{1}{2}}$

$= 3 - 8 \times 6 + 15 \times 2 + 15$

$= 3 - 48 + 30 + 15 = 48 - 48 = 0$

12. Given, $a = 2$ and $b = 3$

(i) $(a^b + b^a)^{-1} = (2^3 + 3^2)^{-1}$

$= (8+9)^{-1} = (17)^{-1} = \frac{1}{17}$

(ii) $(a^a + b^b)^{-1} = (2^2 + 3^3)^{-1}$

$= (4 + 27)^{-1} = 31^{-1} = \frac{1}{31}$

13. LHS $= \frac{x^{p(q-r)}}{x^{q(p-r)}} \div \left(\frac{x^q}{x^p}\right)^r = \frac{x^{p(q-r)}}{x^{q(p-r)}} \times \left(\frac{x^q}{x^p}\right)^{-r}$

$= \frac{x^{pq-pr}}{x^{qp-qr}} \times \left(\frac{x^p}{x^q}\right)^r = \frac{x^{pq-pr} \times x^{pr}}{x^{qp-qr} \times x^{qr}}$

$\left[\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m\right]$

$= \frac{x^{pq-pr+pr}}{x^{qp-qr+qr}} = \frac{x^{pq}}{x^{qp}}$ $(\because a^m \times a^n = a^{m+n})$

$= x^{pq-pq} = x^0$ $\left(\because \frac{a^m}{a^n} = a^{m-n}; a^0 = 1\right)$

$= 1 =$ RHS **Hence proved**

14. $\frac{5^{36}+5^{35}+5^{34}}{5^{32}+5^{31}+5^{30}} + \frac{3^{40}+3^{39}+3^{38}}{3^{41}+3^{40}+3^{39}}$

$= \frac{5^{34}(5^2+5^1+1)}{5^{30}(5^2+5^1+1)} + \frac{3^{38}(3^2+3^1+1)}{3^{39}(3^2+3^1+1)}$

$= \frac{5^{34}}{5^{30}} + \frac{3^{38}}{3^{39}} = 5^{34-30} + 3^{38-39}$

$= 5^4 + 3^{-1}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$= 625 + \frac{1}{3} = \frac{1875+1}{3}$

$= \frac{1876}{3}$

Short Answer Type-II Questions

1.

TR!CK

Between two rational numbers a and b, n rational numbers are given by (a + d), (a + 2d), ..., (a + nd)

where $d = \frac{b-a}{n+1}$

Let $a = 3$ and $b = 4$

Since, six rational numbers are to be found, i.e., $n = 6$

So, $d = \frac{b-a}{n+1} = \frac{4-3}{6+1} = \frac{1}{7}$

1st rational number $= a + d = 3 + \frac{1}{7} = \frac{22}{7}$

2nd rational number $= a + 2d = 3 + \frac{2}{7} = \frac{23}{7}$

3rd rational number $= a + 3d = 3 + \frac{3}{7} = \frac{24}{7}$

4th rational number $= a + 4d = 3 + \frac{4}{7} = \frac{25}{7}$

5th rational number $= a + 5d = 3 + \frac{5}{7} = \frac{26}{7}$

6th rational number $= a + 6d = 3 + \frac{6}{7} = \frac{27}{7}$

Hence, six rational numbers are

$$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7} \text{ and } \frac{27}{7}.$$

2. Let $x = 4.0\overline{35}$ ...(1)

Multiplying eq. (1) by 10, we get

$10x = 40.\overline{35}$ ...(2)

Again, multiplying eq. (2) by 100, we get

$1000x = 4035.\overline{35}$ ...(3)

Subtracting eq. (2) from eq. (3), we get

$990x = 3995$

$\Rightarrow \quad x = \frac{3995}{990} = \frac{799}{198}$

Hence, $4.0\overline{35} = \frac{799}{198}$.

3. $\sqrt{5+2\sqrt{6}} = \sqrt{3+2+2\sqrt{6}}$

$= \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3}.\sqrt{2}}$

$= \sqrt{(\sqrt{3}+\sqrt{2})^2} = \sqrt{3} + \sqrt{2}$

$\sqrt{8-2\sqrt{15}} = \sqrt{5+3-2\sqrt{15}}$

$= \sqrt{(\sqrt{5})^2 + (\sqrt{3})^2 - 2\sqrt{5}.\sqrt{3}}$

$= \sqrt{(\sqrt{5}-\sqrt{3})^2} = \sqrt{5} - \sqrt{3}$

$\therefore \quad \sqrt{5+2\sqrt{6}} + \sqrt{8-2\sqrt{15}}$

$= \sqrt{3} + \sqrt{2} + \sqrt{5} - \sqrt{3}$

$= \sqrt{2} + \sqrt{5}$

4. $\frac{\sqrt{3}+\sqrt{2}}{5+\sqrt{2}} = \frac{\sqrt{3}+\sqrt{2}}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}}$

[Rationalising the denominator]

$= \frac{(\sqrt{3}+\sqrt{2})(5-\sqrt{2})}{5^2 - (\sqrt{2})^2}$

$= \frac{5\sqrt{3}+5\sqrt{2}-\sqrt{6}-2}{25-2}$

$= \frac{5\sqrt{3}+5\sqrt{2}-\sqrt{6}-2}{23}$

5. Given, $a = \frac{\sqrt{2}+1}{\sqrt{2}-1}$

$= \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(\sqrt{2}+1)^2}{(\sqrt{2})^2 - 1^2}$

[Rationalising the denominator]

$= \frac{(\sqrt{2})^2 + 1 + 2\sqrt{2}}{2-1} = \frac{2+1+2\sqrt{2}}{1}$

$= 3 + 2\sqrt{2}$ ...(1)

and $b = \frac{\sqrt{2}-1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{(\sqrt{2}-1)^2}{(\sqrt{2})^2 - 1}$

[Rationalising the denominator]

$= \frac{(\sqrt{2})^2 + 1^2 - 2\sqrt{2}}{2-1} = \frac{2+1-2\sqrt{2}}{1}$

$= 3 - 2\sqrt{2}$ ...(2)

Adding eqs. (1) and (2), we get

$a + b = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6$

and $ab = (3+2\sqrt{2})(3-2\sqrt{2}) = 3^2 - (2\sqrt{2})^2$

$= 9 - 4 \times 2 = 9 - 8 = 1$

Now, $a^2 + b^2 - 4ab = a^2 + b^2 + 2ab - 6ab$

$= (a+b)^2 - 6ab$

$= 6^2 - 6 \times 1 = 36 - 6 = 30$

Alternate Method:

$a^2 + b^2 - 4ab$

$= (3+2\sqrt{2})^2 + (3-2\sqrt{2})^2 - 4(3+2\sqrt{2})(3-2\sqrt{2})$

[From eqs. (1) and (2)]

$= 9 + 8 + 12\sqrt{2} + 9 + 8 - 12\sqrt{2} - 4(3^2 - (2\sqrt{2})^2)$

$= 34 - 4(9-8)$

$= 34 - 4 \times 1 = 30$

6. Given, $\frac{30}{4\sqrt{3}+3\sqrt{2}} = 4\sqrt{3} - a\sqrt{2}$

Rationalising the denominator of LHS,

$\frac{30}{4\sqrt{3}+3\sqrt{2}} \times \frac{4\sqrt{3}-3\sqrt{2}}{4\sqrt{3}-3\sqrt{2}} = 4\sqrt{3} - a\sqrt{2}$

$\Rightarrow \frac{30(4\sqrt{3}-3\sqrt{2})}{(4\sqrt{3})^2 - (3\sqrt{2})^2} = 4\sqrt{3} - a\sqrt{2}$

$\Rightarrow \frac{30(4\sqrt{3}-3\sqrt{2})}{48-18} = 4\sqrt{3} - a\sqrt{2}$

$\Rightarrow \frac{30(4\sqrt{3}-3\sqrt{2})}{30} = 4\sqrt{3} - a\sqrt{2}$

$\Rightarrow 4\sqrt{3} - 3\sqrt{2} = 4\sqrt{3} - a\sqrt{2}$

On comparing both sides, we get

$a = 3$

7. We have $\sqrt{13}$

TiP

Root number must be written as the sum of squares of two natural numbers.

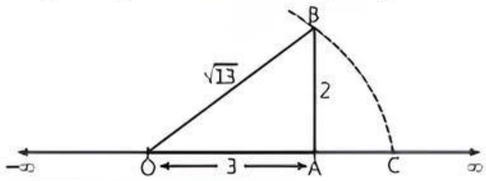
Here, $13 = (3)^2 + (2)^2$

So, we take $a = 3$ and $b = 2$

Firstly, draw a number line and take OA = 3 units on a number line.

Further draw AB = 2 units such that AB $\perp$ OA and join OB.

In right angled ΔOAB, use Pythagoras theorem



$OB = \sqrt{(OA)^2 + (AB)^2}$

$= \sqrt{(3)^2 + (2)^2} = \sqrt{9+4}$

$\sqrt{13}$

Taking O as centre and radius equal to OB, draw an arc, which cuts the number line at C.

Hence, line segment OC represents $\sqrt{13}$ on the number line.

8. Here, $10 = (3)^2 + (1)^2$

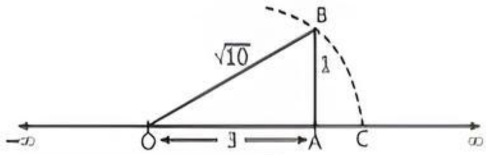
So, we take $a = 3$ and $b = 1$

Firstly, draw a number line and take

OA = 3 units on a number line.

Further draw AB = 1 unit such that $AB \perp OA$ and join OB.

In right angled ΔOAB, use Pythagoras theorem



$OB = \sqrt{(OA)^2 + (AB)^2}$

$= \sqrt{(3)^2 + (1)^2}$

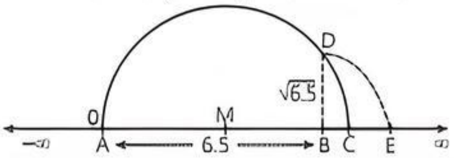
$= \sqrt{9+1} = \sqrt{10}$

Taking O as centre and radius equal to OB, draw an arc, which cuts the number line at C.

Hence, line segment OC represents $\sqrt{10}$ on the number line.

9. Firstly, we draw a line segment AB = 6.5 units on the number line. At point B, take 1 unit distance in right hand side mark as point C.

Let M be the mid-point of AC. Taking M as centre with radius MC (or AM) draw a semi-circle.



Let us draw a perpendicular line BD at point B on the line segment AB, which intersect the semicircle at point D.

$\therefore$ Distance $BD = \sqrt{6.5}$

Finally, draw an arc taking B as centre with radius BD, which intersect the number line at point E.

Hence, point E represents $\sqrt{6.5}$.

10. $\frac{9^{\frac{1}{3}}\times 27^{\frac{-1}{2}}}{3^{\frac{1}{6}}\times 3^{\frac{-2}{3}}} = \frac{9^{\frac{1}{3}}\times 3^{\frac{2}{3}}}{3^{\frac{1}{6}}\times 27^{\frac{1}{2}}} = \frac{(3^2)^{\frac{1}{3}}\times (3)^{\frac{2}{3}}}{(3)^{\frac{1}{6}}\times (3^3)^{\frac{1}{2}}}$

$= \frac{3^{\frac{2}{3}}\times 3^{\frac{2}{3}}}{3^{\frac{1}{6}}\times 3^{\frac{3}{2}}} = \frac{3^{\frac{2}{3}+\frac{2}{3}}}{3^{\frac{1}{6}+\frac{3}{2}}} = \frac{3^{\frac{4}{3}}}{3^{\frac{1+9}{6}}} = \frac{3^{\frac{4}{3}}}{3^{\frac{10}{6}}}$

$= \frac{3^{\frac{4}{3}}}{3^{\frac{5}{3}}} = 3^{\frac{4}{3}-\frac{5}{3}} = 3^{\frac{-1}{3}} = \frac{1}{3^{\frac{1}{3}}}$

11. $\left(\frac{2}{3}\right)^x \cdot \left(\frac{3}{2}\right)^{2x} = \frac{81}{16}$

$\Rightarrow \frac{2^x . 3^{2x}}{3^x . 2^{2x}} = \frac{3^4}{2^4}$

$\Rightarrow \frac{3^{2x-x}}{2^{2x-x}} = \frac{3^4}{2^4}$ $\left[\because \frac{a^m}{a^n} = a^{m-n}\right]$

$\Rightarrow \frac{3^x}{2^x} = \frac{3^4}{2^4} \Rightarrow \left(\frac{3}{2}\right)^x = \left(\frac{3}{2}\right)^4$

Comparing the powers on both sides, we get

$x = 4$

12. $\frac{1}{3}(\sqrt{7})^6 \times (25)^{\frac{3}{2}} \times \left(\frac{1}{5^3}\right)$

$= \frac{1}{3}(7^{\frac{1}{2}})^6 \times (5^2)^{\frac{3}{2}} \times 5^{-3}$

$= \frac{1}{3} \times 7^{\frac{1}{2}\times 6} \times 5^{2\times\frac{3}{2}} \times 5^{-3}$

$= \frac{1}{3} \times 7^3 \times 5^3 \times 5^{-3}$

$= \frac{1}{3} \times 343 \times 5^{(3-3)} = \frac{343}{3} \times 5^0$

$= \frac{343}{3} \times 1 = \frac{343}{3}$

13. $\sqrt[3]{4}, \sqrt{3}, \sqrt[4]{6} = 4^{\frac{1}{3}}, 3^{\frac{1}{2}}, 6^{\frac{1}{4}}$

Taking the LCM of 3, 2 and 4, we get

LCM = 12

$\therefore 4^{\frac{1}{3}}, 3^{\frac{1}{2}}, 6^{\frac{1}{4}} = (4^4)^{\frac{1}{12}}, (3^6)^{\frac{1}{12}}, (6^3)^{\frac{1}{12}}$

$= (256)^{\frac{1}{12}}, (729)^{\frac{1}{12}}, (216)^{\frac{1}{12}}$

Hence, in ascending order, we have

$(216)^{\frac{1}{12}}, (256)^{\frac{1}{12}}, (729)^{\frac{1}{12}}$

or $\sqrt[4]{6}, \sqrt[3]{4}, \sqrt{3}$

14. Given, $\frac{9^n \times 3^2 \times (3)^{\frac{-n}{2} \times -2} - (3^3)^n}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{2n} \times 3^2 \times 3^n - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27} \Rightarrow \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{3^{3n}(3^2 - 1)}{3^{3m} \times 2^3} = \frac{1}{27}$

$\Rightarrow \frac{8 \times 3^{3n}}{8 \times 3^{3m}} = \frac{1}{3^3}$

$\Rightarrow \quad 3^{3n-3m} = 3^{-3}$

Comparing the exponents of 3, we get

$3n - 3m = -3$

$\Rightarrow \quad n - m = -1$

or $\quad m - n = 1$ **Hence proved**

15. LHS $= \dfrac{1}{1+x^{a-b}} + \dfrac{1}{1+x^{b-a}} = \dfrac{1}{1+\dfrac{x^a}{x^b}} + \dfrac{1}{1+\dfrac{x^b}{x^a}}$

$= \dfrac{x^b}{x^b+x^a} + \dfrac{x^a}{x^a+x^b}$

$= \dfrac{x^b+x^a}{x^b+x^a} = 1 = \text{RHS}$ **Hence proved**

Long Answer Type Questions

1. Let the two rational numbers be $m = \frac{3}{5}$ and $n = \frac{8}{3}$.

(i) Sum $= \frac{3}{5} + \frac{8}{3} = \frac{9+40}{15} = \frac{49}{15}$ [Rational]

(ii) Difference $= \frac{8}{3} - \frac{3}{5} = \frac{40-9}{15} = \frac{31}{15}$ [Rational]

(iii) Product $= \frac{8}{3} \times \frac{3}{5} = \frac{8}{5}$ [Rational]

(iv) Division $= \frac{8}{3} \div \frac{3}{5} = \frac{40}{9}$ [Rational]

2. Let $x = 1.3\overline{2} = 1.32222......$ ...(1)

Multiplying eq. (1) by 10, we get

$10x = 13.222...$ ...(2)

Again, multiplying eq. (2) by 10, we get

$100x = 132.222...$ ...(3)

Subtracting eq. (2) from eq. (3), we get

$100x - 10x = (132.222...) - (13.222...)$

$\Rightarrow \quad 90x = 119 \quad \Rightarrow \quad x = \frac{119}{90}$

Again, $\quad y = 0.\overline{35} = 0.353535...$ ...(4)

Multiplying eq. (4) by 100, we get

$100y = 35.3535...$ ...(5)

Subtracting eq. (4) from eq. (5), we get

$100y - y = (35.3535...) - (0.353535...)$

$\Rightarrow \quad 99y = 35 \quad \Rightarrow \quad y = \frac{35}{99}$

Now, $\quad 1.3\overline{2} + 0.\overline{35} = x + y = \frac{119}{90} + \frac{35}{99}$

$= \frac{1309+350}{990} = \frac{1659}{990} = \frac{553}{330}$

3. No, xy is necessarily an irrational only when $x \neq 0$.

Let x be a non-zero rational and y be an irrational. Then, we have to show that xy be an irrational.

If possible, let xy be a rational number. Since, quotient of two non-zero rational numbers is a rational number.

So, $\left(\frac{xy}{x}\right)$ is a rational number.

$\Rightarrow y$ is a rational number.

But, this contradicts the fact that y is an irrational number. So, our supposition is wrong.

Hence, xy is an irrational number. But, when $x = 0$, then $xy = 0$, which is a rational number.

4. LHS $= \dfrac{1}{1+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + ... + \dfrac{1}{\sqrt{8}+3}$

$= \dfrac{1}{1+\sqrt{2}} \times \dfrac{1-\sqrt{2}}{1-\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} \times \dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$

$+ \dfrac{1}{\sqrt{3}+\sqrt{4}} \times \dfrac{\sqrt{3}-\sqrt{4}}{\sqrt{3}-\sqrt{4}} + ...$

$+ \dfrac{1}{\sqrt{8}+3} \times \dfrac{\sqrt{8}-3}{\sqrt{8}-3}$

[Rationalising the denominator of each part]

$= \dfrac{1-\sqrt{2}}{1-2} + \dfrac{\sqrt{2}-\sqrt{3}}{2-3} + \dfrac{\sqrt{3}-\sqrt{4}}{3-4} + ... + \dfrac{\sqrt{8}-3}{8-9}$

$= \dfrac{1-\sqrt{2}}{-1} + \dfrac{\sqrt{2}-\sqrt{3}}{-1} + \dfrac{\sqrt{3}-\sqrt{4}}{-1} + ... + \dfrac{\sqrt{8}-3}{-1}$

$= -1 + \sqrt{2} - \sqrt{2} + \sqrt{3} - \sqrt{3} + \sqrt{4} - ... - \sqrt{8} + 3$

$= -1 + 3 = 2$ [Only first and last terms are left]

$=$ RHS

COMMON ERROR

Some of the students make a mistake of sum of these terms in last 3rd line. So please be avoid to do a mistake.

5. Given, $\dfrac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}-\sqrt{3}} + \dfrac{2\sqrt{5}-\sqrt{3}}{2\sqrt{5}+\sqrt{3}} = a + \sqrt{15}\,b$

$\Rightarrow \quad \dfrac{(2\sqrt{5}+\sqrt{3})^2 + (2\sqrt{5}-\sqrt{3})^2}{(2\sqrt{5}-\sqrt{3})(2\sqrt{5}+\sqrt{3})} = a + \sqrt{15}\,b$

TR!CK

Using identities

(i) $(a+b)^2 = a^2 + b^2 + 2ab$

(ii) $(a-b)^2 = a^2 + b^2 - 2ab$

(iii) $(a+b)(a-b) = a^2 - b^2$

$\Rightarrow \quad \dfrac{(2\sqrt{5})^2 + (\sqrt{3})^2 + 2\times(2\sqrt{5})(\sqrt{3}) + (2\sqrt{5})^2 + (\sqrt{3})^2 - 2\times(2\sqrt{5})(\sqrt{3})}{(2\sqrt{5})^2 - (\sqrt{3})^2} = a + \sqrt{15}\,b$

$\Rightarrow \quad \dfrac{20+3+4\sqrt{15}+20+3-4\sqrt{15}}{20-3} = a + \sqrt{15}\,b$

$\Rightarrow \quad \dfrac{46}{17} = a + \sqrt{15}\,b$

Comparing rational and irrational parts, we get

$a = \frac{46}{17}$ and $b = 0$

COMMON ERROR

Some of the students commit mistake, while comparing rational and irrational parts. So, they need more practice of these type of questions.

6. $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} = \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} \times \frac{\sqrt{10}-\sqrt{3}}{\sqrt{10}-\sqrt{3}}$

[Rationalising the denominator]

$= \frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{(\sqrt{10})^2-(\sqrt{3})^2} = \frac{7(\sqrt{30}-3)}{10-3}$

$= \frac{7(\sqrt{30}-3)}{7} = \sqrt{30}-3$ ...(1)

$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} \times \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}-\sqrt{5}} = \frac{2\sqrt{30}-2\times 5}{(\sqrt{6})^2-(\sqrt{5})^2}$

[Rationalising the denominator]

$= \frac{2\sqrt{30}-10}{6-5} = 2\sqrt{30}-10$ ...(2)

$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} \times \frac{\sqrt{15}-3\sqrt{2}}{\sqrt{15}-3\sqrt{2}}$

[Rationalising the denominator]

$= \frac{3\sqrt{30}-18}{15-18} = \frac{3\sqrt{30}-18}{-3}$

$= \frac{3(\sqrt{30}-6)}{-3} = -(\sqrt{30}-6) = 6-\sqrt{30}$ ...(3)

Now, $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$

$= \sqrt{30}-3-(2\sqrt{30}-10)-(6-\sqrt{30})$

[From eqs. (1), (2) and (3)]

$= \sqrt{30}-3-2\sqrt{30}+10-6+\sqrt{30}$

$= 10-9+2\sqrt{30}-2\sqrt{30} = 1$

7. Given, $x = \frac{1}{2-\sqrt{3}}$

Rationalising the denominator, we get

$x = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3}$

Now, $x-2 = \sqrt{3}$

Squaring both sides, we get

$(x-2)^2 = (\sqrt{3})^2 = 3$

$\Rightarrow x^2-4x+4 = 3$

$\Rightarrow x^2-4x+1 = 0$

$\therefore x^3-2x^2-7x+5 = x(x^2-4x+1) + 2(x^2-4x+1)+3$

$= x\times 0+2\times 0+3 = 3$

Hence, the value of given expression is 3.

8. Given, $x = \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}}$

$x = \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}-\sqrt{p-q}} \times \frac{\sqrt{p+q}+\sqrt{p-q}}{\sqrt{p+q}+\sqrt{p-q}}$

(Rationalising the denominator)

$= \frac{(\sqrt{p+q}+\sqrt{p-q})^2}{(\sqrt{p+q})^2-(\sqrt{p-q})^2}$

$\Rightarrow x = \frac{p+q+p-q+2\times\sqrt{p+q}\times\sqrt{p-q}}{(p+q)-(p-q)}$

$= \frac{2p+2\sqrt{p^2-q^2}}{2q}$

$\Rightarrow x = \frac{p+\sqrt{p^2-q^2}}{q} \Rightarrow qx = p+\sqrt{p^2-q^2}$

$\Rightarrow qx-p = \sqrt{p^2-q^2}$

Squaring both sides, we get

$(qx-p)^2 = (\sqrt{p^2-q^2})^2$

$\Rightarrow q^2x^2+p^2-2pqx = p^2-q^2$

$\Rightarrow q^2x^2-2pqx+q^2 = 0$

$\Rightarrow q(qx^2-2px+q) = 0$

$\Rightarrow qx^2-2px+q = 0$ $(\because q \neq 0)$

Hence proved.

9. We have, $a^2+ab+b^2 = (a+b)^2-ab$ ...(1)

and $a^2-ab+b^2 = (a-b)^2+ab$ ...(2)

Given, $a = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ and $b = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

Rationalising the denominator, we get

$a = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{(\sqrt{5}+\sqrt{2})^2}{5-2}$

$= \frac{5+2+2\sqrt{10}}{3} = \frac{7+2\sqrt{10}}{3}$

and $b = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} \times \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{(\sqrt{5}-\sqrt{2})^2}{5-2}$

$= \frac{5+2-2\sqrt{10}}{3} = \frac{7-2\sqrt{10}}{3}$

Now, $(a+b)^2-ab = \left(\frac{7+2\sqrt{10}}{3}+\frac{7-2\sqrt{10}}{3}\right)^2 - \left(\frac{7+2\sqrt{10}}{3}\times\frac{7-2\sqrt{10}}{3}\right)$

$= \left(\frac{14}{3}\right)^2 - \left(\frac{49-40}{9}\right)$

$= \frac{196}{9}-\frac{9}{9} = \frac{187}{9}$

$(a-b)^2+ab = \left(\frac{7+2\sqrt{10}}{3}-\frac{7-2\sqrt{10}}{3}\right)^2 + \left(\frac{7+2\sqrt{10}}{3}\times\frac{7-2\sqrt{10}}{3}\right)$

$= \left(\frac{4\sqrt{10}}{3}\right)^2 + \left(\frac{49-40}{9}\right) = \frac{160}{9}+\frac{9}{9} = \frac{169}{9}$

$\therefore \frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{(a+b)^2-ab}{(a-b)^2+ab}$

[From eqs. (1) and (2)]

$= \frac{\frac{187}{9}}{\frac{169}{9}} = \frac{187}{169}$

10. $\frac{4}{(216)^{-\frac{2}{3}}}+\frac{1}{(256)^{-\frac{3}{4}}}+\frac{2}{(243)^{-\frac{1}{5}}}$

$= 4(216)^{\frac{2}{3}}+(256)^{\frac{3}{4}}+2(243)^{\frac{1}{5}}$

$= 4(6^3)^{\frac{2}{3}}+(4^4)^{\frac{3}{4}}+2(3^5)^{\frac{1}{5}}$

$= 4\times 6^2+4^3+2\times 3$

$= 4\times 36+64+6$

$= 144+64+6$

$= 214$

11. $\left(\frac{2^{-1}\times 3^2}{2^2\times 3^{-4}}\right)^{\frac{7}{2}}\times\left(\frac{2^{-2}\times 3^3}{2^3\times 3^{-5}}\right)^{\frac{-5}{2}}$

$= \left(\frac{3^6}{2^3}\right)^{\frac{7}{2}}\times\left(\frac{3^8}{2^5}\right)^{\frac{-5}{2}}$ $\quad\left[\because \frac{a^m}{a^n}=a^{m-n}\right]$

$= \frac{3^{6\times 7/2}}{2^{3\times 7/2}}\times\frac{3^{8\times -5/2}}{2^{5\times -5/2}}$

$= \frac{3^{21}}{2^{21/2}}\times\frac{3^{-20}}{2^{-25/2}}=\frac{3^{21}\times 3^{-20}}{2^{21/2}\times 2^{-25/2}}$

$= \frac{3^{21-20}}{2^{\frac{21-25}{2}}}=\frac{3^1}{2^{-4/2}}=3\times 2^2=3\times 4=12$

12. Given, $a=\frac{2^{x-1}}{2^{x-2}}, b=\frac{2^{-x}}{2^{x+1}}$ and $a-b=0$

$\Rightarrow \frac{2^{x-1}}{2^{x-2}}-\frac{2^{-x}}{2^{x+1}}=0$

$\Rightarrow 2^{x-1-(x-2)}-2^{-x-(x+1)}=0$

$\Rightarrow 2^1-2^{-2x-1}=0$

$\Rightarrow 2^{-2x-1}=2^1$

Comparing the exponents of 2, we get

$-2x-1=1$

$\Rightarrow 2x=-2$

$\Rightarrow x=-1$

Hence, the value of x is -1.

13. Given, $5^{2x-1}-25^{x-1}=2500$

$\Rightarrow 5^{2x-1}-5^{2(x-1)}=2500$

$\Rightarrow 5^{2x-2+1}-5^{2x-2}=2500$

$\Rightarrow 5^{2x-2}(5-1)=2500$

$\Rightarrow 5^{2x-2}=\frac{2500}{4}=625=5^4$

Comparing the exponents of 5, we get

$2x-2=4$

$\Rightarrow 2x=6$

$\Rightarrow x=3$

Chapter Test

Multiple Choice Questions

Q 1. A rational number between 7 and 11 is:

a. 7 b. $\sqrt{8}$

c. 9 d. $\sqrt{6}$

Q 2. The rationalisation factor of $8+\sqrt{11}$ is:

a. 53 b. $\frac{1}{8+\sqrt{11}}$

c. $\sqrt{11}-8$ d. $8-\sqrt{11}$

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4) *In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

c. Assertion (A) is true but Reason (R) is false.

d. Assertion (A) is false but Reason (R) is true.

Q 3. Assertion (A): The simplified form of $11^3\times 11^8$ is 11^{24}

Reason (R): If $a>0$ be a real number and p and q be rational numbers, then $a^p\times a^q=a^{p+q}$.

Q 4. Assertion (A): The sum of two irrational numbers $2-\sqrt{11}$ and $2+\sqrt{11}$ is rational number.

Reason (R): The sum of two irrational numbers is always an irrational number.

Fill in the Blanks

Q 5. If the decimal representation of a number is non-terminating non-repeating, then the number is a/an .... number.

Q 6. The value of $\sqrt[4]{(16)^{-3}}$ is ....

True/False

Q 7. The product of $(5-\sqrt{7})$ and $(5+\sqrt{7})$ is a/an rational number.

Q 8. For rationalising the denominator of the expression $\frac{1}{\sqrt{18}}$, we multiply and divide by $\sqrt{2}$.

Case Study Based Questions

Q 9. In a classroom, one day a math teacher taught students of class IX about the number systems. She drew a number line and told them that the number line represents various types of number on it. A number of the form $\frac{p}{q}$, $q \neq 0$ is a rational number, which can be represented on a number line.

On the basis of the above information, solve the following questions:

(i) Find a rational number between 3 and 7.

(ii) Find the decimal number of $\frac{5}{15}$.

OR

Write the rational form of $3.\overline{5}$.

(iii) If $x+\sqrt{3}=7$, then find the value of $\frac{1}{x}$.

Very Short Answer Type Questions

Q 10. Simplify $\sqrt{108}+\sqrt{1200}-\sqrt{27}$.

Q 11. Write the simplified value of:

$$(36)^{-\frac{1}{2}}+(36)^{\frac{1}{2}}$$

Short Answer Type-I Questions

Q 12. Find two irrational numbers between $\frac{1}{11}$ and $\frac{3}{11}$.

Q 13. Find the value of a and b, if

$$\frac{6+\sqrt{5}}{6-\sqrt{5}}=a+b\sqrt{5}.$$

Short Answer Type-II Questions

Q 14. Show how $\sqrt{29}$ can be represented on the number line?

Q 15. Find x, if $\left(\frac{5}{7}\right)^{x}\left(\frac{7}{5}\right)^{2x}=\frac{343}{125}$.

Long Answer Type Question

Q 16. Simplify $\frac{1}{2\sqrt{3}}+\frac{2}{\sqrt{5}-\sqrt{3}}+\frac{1}{2-\sqrt{5}}$